

B.Sc Part I (Physics Hons)
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Question:- What is Doppler's effect. Discuss relativistically the phenomenon of Doppler effect in light.

Ans Doppler Effect :->

An apparent change in frequency of received radiation due to relative motion between source, observer and medium is known Doppler effect.

Let us consider a plane wave whose wave motion normal points in direction given by direction cosines l, m and n w.r to an observer in the frame S . Let its phase velocity be u such that its components are u_l, u_m & u_n . let its wavelength, frequency and Time period be λ, ν and T respectively such that $\lambda = \frac{u}{\nu} = uT$. These quantities in frame S' are written with primed symbols. Let us take an equation of plane wave as

$$\psi = A \exp \left[2\pi i \left(\frac{lx + my + nz}{\lambda} - \frac{t}{T} \right) \right] \quad \text{--- (1)}$$

$$= A \exp \left[\vec{k} \cdot \vec{x} - \frac{2\pi i t}{T} \right] \quad \text{--- (2)}$$

Here vector $\vec{k} = \frac{2\pi}{\lambda} (l, m, n)$ and points in the direction of wave propagation and $\vec{x} = (x, y, z)$

Now let us consider a set of four quantities

$$(K_i) = \left(\vec{k} \cdot \frac{i\omega}{c} \right), \quad \because \omega = \frac{2\pi}{T} = 2\pi\nu$$

$$\text{from eq}^n \text{(1)} \quad \psi = A \exp(K_i x_i) \quad \text{--- (3)}$$

where x_i is a four position vector as defined subsequently in four vectors. The inner product of (K_i) with four vector (x_i) gives a scalar quantity, hence (K_i) is also a four vector with following usual spatial and temporal transformations as given in derivation of Lorentz transformations relations.

$$\therefore \vec{k}' = \vec{k} + \frac{(\vec{k} \cdot \vec{v}) \vec{v}}{v^2} (\beta - 1) - \beta \vec{v} \frac{\omega}{c^2} \quad \text{--- (4)}$$

$$\text{and } \omega' = \beta \left[\omega - (\vec{k} \cdot \vec{v}) \right] \quad \text{Her } \beta = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{--- (5)}$$



Eqⁿ ①, ③ and ④ gives the direction of propagation in frame S' and S'' while eqⁿ ⑤ inter-related the frequencies in S' and S'' . Since

$$\vec{k} = \frac{2\pi}{\lambda} \cdot \frac{\vec{u}}{u} = 2\pi \frac{2\pi}{u^2}$$

from eqⁿ ⑤ $\omega' = 2\pi \nu' = \beta [2\pi \nu - 2\pi \nu \frac{(\vec{u} \cdot \vec{v})}{u^2}]$

$$\Rightarrow \nu' = \beta \nu \left[1 - \frac{(\vec{u} \cdot \vec{v})}{u^2} \right] \dots \dots \dots \textcircled{6}$$

This gives the Relativistic Doppler's effect and is different from classical case due to the factor

$$\beta = \frac{1}{\sqrt{1 - v^2/c^2}}$$

If we apply it to the case of light propagating in free space, we $u = c$ and for the source system attached to the frame S'' we get $\nu' = \nu_0$

Hence from eqⁿ ⑥

$$\nu_0 = \nu \beta \left[1 - \frac{\vec{c} \cdot \vec{v}}{c^2} \right]$$

$$\text{or, } \nu = \frac{\nu_0 \sqrt{1 - v^2/c^2}}{\left[1 - \frac{\vec{c} \cdot \vec{v}}{c^2} \right]} \dots \dots \dots \textcircled{7}$$

In non relativistic case where $\frac{v^2}{c^2} \ll 1$, we get

$$\nu = \frac{\nu_0}{\left[1 - \frac{\vec{c} \cdot \vec{v}}{c^2} \right]} \dots \dots \dots \textcircled{8}$$

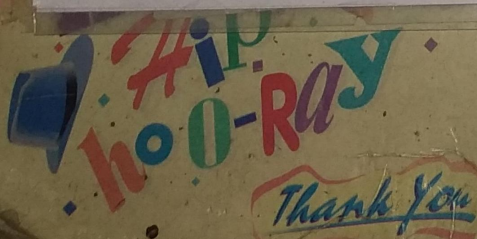
This is the usual classical relation.

The following cases of interest may be studied with the help of eqⁿ ⑦

(i) In case the source of light moves radially away from the observer, we have $\vec{c} \cdot \vec{v} = -cv$, in this case

eqⁿ ⑦ becomes

$$\nu = \nu_0 \sqrt{\frac{c-v}{c+v}} \dots \dots \dots \textcircled{9}$$



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which shows that $v_0 \gg v$, However in classical case in which $\frac{v^2}{c^2} \ll 1$, we get from (7)

$$v = v_0 \left[\frac{c}{c+v} \right] \text{ --- --- --- --- --- (10)}$$

(ii) In alternate case in which the source moves radially towards the observer, we have

$$\vec{r} \cdot \vec{v} = cv \text{ and from eq}^n \text{ (7)}$$

$$v = v_0 \sqrt{\frac{c+v}{c-v}} \text{ --- --- --- --- --- (11)}$$

which shows that $v > v_0$ in such a case, However in classical case when $\frac{v^2}{c^2} \ll 1$

$$\text{from eq}^n \text{ (7) } v = v_0 \left[\frac{c}{c-v} \right] \text{ --- --- --- --- --- (12)}$$

(iii) In case when the source moves at right angle to the observer, $\vec{r} \cdot \vec{v} = 0$,

from eqⁿ (7)

$$v = v_0 \sqrt{1 - \frac{v^2}{c^2}} \text{ --- --- --- --- --- (13)}$$

This shows that no transverse Doppler effect exists in classical case while it exists in relativistic case only with $v_0 > v$.

